Mathematics

Foundation

Unit 1

Multiplying Whole Numbers

Note: 12 x 3 is the same as 3 x 12

Example 1 – short multiplication: 127 x 6

$$\begin{array}{r} 1\ 2\ 7\ \times \\ 6\ \ \\ \hline 7\ 6\ 2\ \ \\ \hline {}_{1\ \ 4}\ \ \ \ \end{array}$$

1st work out 7 × 6 = 42 Write down the 2 units in the box and carry the 4 tens under the tens column

Next work out 2 × 6 = 12 (or 20 × 6) you need to add on the carry of 4 (or 40) to make 16 (or 160). Place 6 (or 6 tens) in the box in the tens column and carry the 1(or 100) in the hundreds column

Lastly, work out 1 × 6 (or 100 × 6) and add on the carry of 1 (or 100) to make 7 (or 700). Place 7 in the box in the hundreds column

Example 2 – long multiplication: 352 x 27

Method 1: Column Method

$$\begin{array}{r} 3\ 5\ 2\ \times \\ 2\ 7\ \ \\ \hline 2\ 4\ 6\ 4\ + \\ 7\ 0\ 4\ 0\ \ \\ \hline 9\ 5\ 0\ 4\ \ \\ \hline {}_{1}\ \ \ \ \ \ \ \end{array}$$

352 x 7 = 2464

Put a 0 in the ones column as we are now multiplying by the number in the tens column 352 x 2 = 704

Add the two rows, 2464 + 7040 = 950

Example 3 – long multiplication: 23 x 34

Method 2: Grid Method

23 is split into 20 and 3

34 is split into 30 and 4

x	20	3
30	600	90
4	80	12

30 x 20 = 600

30 x 3 = 90

4 x 20 = 80

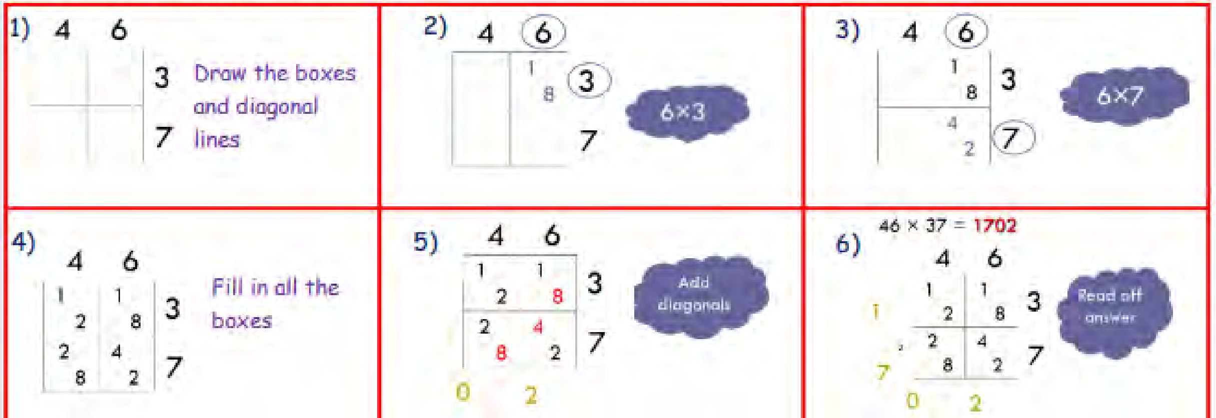
4 x 3 = 12

Add all the answers

600 + 90 + 80 + 12 = 782

Example 4 – long multiplication: 46 x 37

Method 3: Box Method



Draw the boxes and diagonal lines

Fill in all the boxes

46 × 37 = 1702

Mathematics

Foundation

Unit 1

Multiplying and Dividing by Multiples of 10:

When you multiply your digits move left, and when you divide your digits move right. The distance they move depends on the amount of zeros in your number (10, 100, 1000 …). Eg. If you are multiplying by 100 the digits move to the left 2 places because 100 has 2 zeros.

Example 1: 43 x 10 = 430

43 moves one place value to the left (10 has one zero) and the space is filled in with a zero

Example 2: 789 x 1000 = 789000

789 moves three place values to the left (1000 has three zeros) and the spaces are filled in with zeros

Example 3: 3200 ÷ 100 = 32

3200 moves two place values to the right (100 has two zeros),

Example 4: 86 ÷ 10 = 8.6

86 moves one place value to the right (10 has one zero), the 6 moves into the tenths column, the answer is a decimal

BODMAS / BIDMAS

Remember, it must be used like this:

First do any: (Brackets)

Followed by any: Indices

Left to right do any: D÷vision & M×ltiplication

Lastly, left to right: Addi+ion & Subtr-ction

BIDMAS / BODMAS:

BIDMAS or BODMAS is a way of helping you to remember the order in which to do your calculations.

Example 1: $2 + \underline{7 \times 10}$

$= 2 + 70$

$= 72$

This question involves addition and multiplication, using the rules of BIDMAS, multiplication is first

Example 2: $\underline{22 - 6} + 4$

$= 16 + 4$

$= 20$

This question involves subtraction and addition, using the rules of BIDMAS, work left to right doing whatever is first

Example 3: $\underline{(4 + 5)}^2 - 4 \times 9$

$= \underline{(9)^2} - 4 \times 9$

$= 81 - \underline{4 \times 9}$

$= 81 - 36$

$= 45$

This question involves multiple operations, just follow the rules of BIDMAS. Brackets first, then indices, then multiplying, then subtraction

Mathematics

Foundation

Unit 2

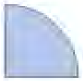
What angle is ...

... a full turn?

360°

... a half turn?

180°



... a quarter turn?

90°

... a three quarter turn?

270°

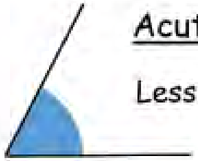
What type of angle is ...

Right-Angle

Exactly 90°

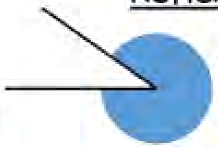
Obtuse Angle

Greater than 90° and less than 180°



Acute Angle

Less than 90°

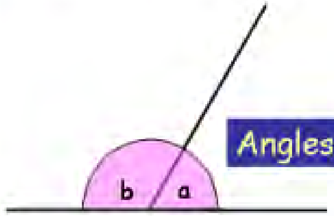


Reflex Angle

Greater than 180° and less than 360°

Angles on a Straight Line

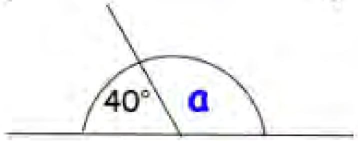
Fact: Angles on a straight line add up to 180°



Angles a + b = 180°

How to spot it: Find any continuous straight line, with another straight line joining it or cutting across it

Example 1: Find the size of angle a



Angles on a straight line add to 180°

$a = 180 - 40$

$a = 140°$

Example 2: Find the size of angle x

Angles on a straight line add to 180°

$90 + 30 = 120$

$x = 180 - 120$

$x = 60°$

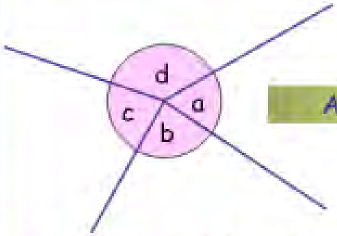
Mathematics

Foundation

Unit 2

Angles around a Point

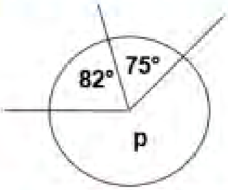
Fact: Angles around a point add up to 360°



Angle a + b + c + d = 360°

How to spot it: If you have a collection of lines all crossing at one point, then it is time to use this rule.

Example 1: Find the size of angle p



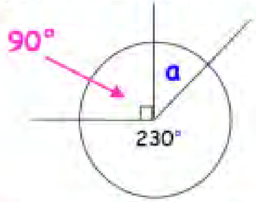
Angles around a point add to 360°

$82 + 75 = 157$

$p = 360 - 157$

$p = 203^\circ$

Example 2: Find the size of angle a



Angles around a point add to 360°

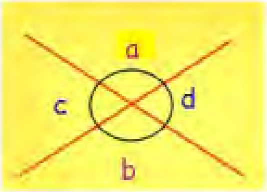
$90 + 230 = 320$

$a = 360 - 320$

$a = 40^\circ$

Opposite Angles

Fact: Opposite Angles are equal



a = b

c = d

How to spot it: Find two continuous straight lines crossing at a point. The pairs of angles opposite each other will be equal

Note: All the angles around that point will add up to 360°

Example 1: Find the size of angles m and n

Opposite angles are equal

$m = 145^\circ$

Angles on a straight line add to 180°

$n = 180 - 145$

$n = 35^\circ$

Mathematics

Foundation

Unit 3

Factors

The Factors of a number are all the whole numbers (integers) that divide into your number exactly (there must not be a remainder).

For example: The factors of 12 are: 1, 2, 3, 4, 6 and 12, the factors of 55 are: 1, 5, 11, and 55

Note: 1 is a factor of all numbers, and so is the number itself.

Multiples

The Multiples of a number are all the numbers in the number's times table.

For example: The multiples of 2 are all the numbers in the 2 times table (2, 4, 6, 8, 10, ...), the first three multiples of 6 are 6, 12, 18.

Reciprocals

To find the reciprocal of a whole number, turn it into a fraction by putting 1 over the number.

For example: The reciprocal of 7 would be $\frac{1}{7}$.

The reciprocal of 35 would be $\frac{1}{35}$.

To find the reciprocal of a fraction, flip the fraction upside down.

For example: The reciprocal of $\frac{3}{4}$ would be $\frac{4}{3}$.

The reciprocal of $\frac{1}{5}$ would be $\frac{5}{1} = 5$.

Index Form / Indices

Indices are just another word for "power".

The base $\longrightarrow 5^4 \longleftarrow$ The index or power

For example:

$3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5.

Five 3's multiplied together, so the index number is 5

2^4 means $2 \times 2 \times 2 \times 2$

Index number is 4, so four 2's multiplied together

For example: Calculate the value of 2^4.

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 4 \times 2 \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

For example: Calculate the value of $10^4 \times 2^3$.

$$\begin{aligned} 10^4 &= 10 \times 10 \times 10 \times 10 \\ &= 100 \times 10 \times 10 \\ &= 1000 \times 10 \\ &= 10000 \end{aligned}$$

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

Work each one out separately, then multiply the answers

So, $10^5 \times 2^3 = 10000 \times 8 = 80000$

Mathematics

Foundation

Unit 5

Finding a Fraction of a Quantity

To find a fraction of a quantity we use the rule: "Divide by the bottom, times by the top"

This means we divide the quantity by the denominator, then multiply the answer by the numerator.

Example 1- Non-Calculator: Calculate $\frac{3}{4}$ of 20

Divide by the bottom: $20 \div 4 = 5$

Multiply by the top: $5 \times 3 = 15$

So, $\frac{3}{4}$ of 20 is 15

Make sure to write down your workings, either like above, or in one step $20 \div 4 \times 3 = 15$.

Example 2 - Calculator: Calculate $\frac{5}{7}$ of 17.5

You can type this straight into a calculator

$17.5 \div 7 \times 5 = 12.5$

So, $\frac{5}{7}$ of 17.5 is 12.5

Example 3: Sam comes from a large family. He has 80 relatives altogether, who live in Canada, Japan, and Wales. $\frac{1}{5}$ of his relatives live in Canada. $\frac{3}{8}$ of his relatives live in Japan. The rest of his relatives live in Wales. How many relatives live in Wales?

Work out how many relatives live in Canada:	$\frac{1}{5}$ of 80	$80 \div 5 = 16$		16 relatives live in Canada
Work out how many relatives live in Japan:	$\frac{3}{8}$ of 80	$80 \div 8 = 10$	$10 \times 3 = 30$	30 relatives live in Japan
Work out how many relatives are left:	$80 - 16 - 30 = 34$			34 relatives live in Wales

Mathematics

Foundation

Unit 5

Multiplying Fractions

To multiply fractions:

- Multiply both numerators;
- multiply both denominators;
- simplify the answer if possible, or cancel down within the question before multiplying

Example 1: $\frac{2}{5}\times\frac{5}{8}$ (simplifying the answer)

Numerators multiplied, 2 x 5 = 10

Denominators multiplied, 5 x 8 = 40

$$\frac{2}{5}\times\frac{5}{8}=\frac{10}{40}\overset{\div 10}{\underset{\div 10}{=}}\frac{1}{4}$$

Example 2: $\frac{2}{5}\times\frac{5}{8}$ (simplifying within the question)

Cancel down any numerator and denominator. This means cancel down 2 and 8 by dividing by 2. Then cancel down both 5s by dividing by 5.

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}}=\frac{1}{4}$$

Example 3: $\frac{4}{7}\times 3$ (multiplying a fraction by a whole number)

Write the 3 as a fraction by putting it over 1. Then multiply as above.

$$\frac{4}{7}\times\frac{3}{1}=\frac{12}{7}=1\frac{5}{7}$$

Dividing Fractions

To divide fractions:

- Keep the first fraction the same;
- change the sign from a divide to a multiply;
- flip the second fraction upside down
- continue as you would for multiplying fractions

Example 1: $\frac{3}{4}\div\frac{5}{16}$ (dividing a fraction by a fraction)

$$\frac{3}{4}\div\frac{5}{16}=\frac{3}{4}\times\frac{16}{5}$$

$$=\frac{48}{20}\overset{\div 4}{\underset{\div 4}{=}}\frac{12}{5}=2\frac{2}{5}$$

Example 2: $\frac{9}{15}\div 3$ (dividing a fraction by a whole number)

$$\frac{9}{15}\div\frac{3}{1}=\frac{9}{15}\times\frac{1}{3}$$

Write the 3 as a fraction by putting it over 1. Then continue as above.

$$=\frac{9}{45}\overset{\div 3}{\underset{\div 3}{=}}\frac{3}{15}\overset{\div 3}{\underset{\div 3}{=}}\frac{1}{5}$$

Mathematics

Foundation

Unit 8

Percentages

A percentage is just a fraction whose denominator (bottom) is 100.
So, if we say "32%", what we mean is $\frac{32}{100}$ or 32 out of 100.

Percentage of an Amount – Non-Calculator

Method We can calculate all percentages by first calculating some of these:

Example: You have £320. Find (a) 15%, (b) 63%, (c) 17.5%

Start by writing down the percentages that you know which might help:

To find 10% → Divide by 10 → 320 ÷ 10 = 32 → 10% = £32
To find 1% → Divide by 100 → 320 ÷ 100 = 3.2 → 1% = £3.20
To find 50% → Divide by 2 → 320 ÷ 2 = 160 → 50% = £160
To find 20% → Double 10% → 32 x 2 = 64 → 20% = £64
To find 5% → Half 10% → 32 ÷ 2 = 16 → 5% = £16
To find 2.5% → Half 5% → 16 ÷ 2 = 8 → 2.5% = £8

You can build up your answers with a bit of simple addition.

(a) 15%

15% = 10% + 5%
= £32 + £16
= £48

(b) 63%

63% = 50% + 10% + 1% + 1% + 1%
= £160 + £32 + £3.20 + £3.20 + £3.20
= £201.60

(c) 17.5%

17.5% = 10% + 5% + 2.5%
= £32 + £16 + £8
= £56

Percentage of an Amount - Calculator

Finding a percentage of an amount using a calculator can be done in one easy step.

Example: Find 23% of 135g (23% percent can be written as 23 ÷ 100, or $\frac{23}{100}$)

Type into the calculator: $23 \div 100 \times 135 =$

Make sure you write the workings down as well as the answer

$$23 \div 100 \times 135 = 31.05g$$

One Number as a Percentage of Another

Example Non-Calculator:

Write 19 as a percentage of 25?

Step 1: Write as a fraction and multiply it by 100

$$\frac{19}{25} \times 100$$

Step 2: Multiply (look back at Unit 5 to recall how to multiply a fraction by a whole number)

$$\frac{19}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1} = 76\%$$

(19 is 76% of 25)

Example Calculator:

Write 256 as a percentage of 780?

Step 1: Type into the calculator

256 ÷ 780 x 100 =

Make sure you write the workings down as well as the answer.

256 ÷ 780 x 100 = 32.82% (2 dp)

(256 is 32.82% of 780)

Mathematics

Foundation

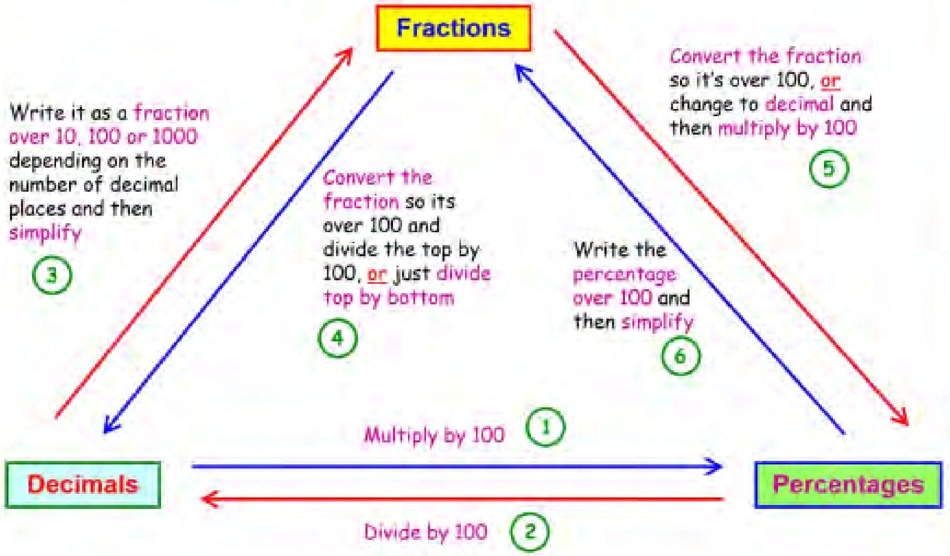
Unit 10

Fractions, Decimals and Percentages

Fractions, Decimals and Percentages are all closely related to each other, and you need to be comfortable changing between each of them.

You can use this diagram to help you.

Follow the arrows depending on what you need to change and follow the numbers for the examples.



Fractions

Decimals

Percentages

Write it as a fraction over 10, 100 or 1000 depending on the number of decimal places and then simplify ③

Convert the fraction so its over 100 and divide the top by 100, or just divide top by bottom ④

Convert the fraction so it's over 100, or change to decimal and then multiply by 100 ⑤

Write the percentage over 100 and then simplify ⑥

Multiply by 100 ①

Divide by 100 ②

Examples:

① What is 0.364 as a percentage?

Just multiply by 100 and be careful with the decimal point!

0.364 × 100

= 36.4%

② Convert 8.3% into a decimal

Just divide by 100 and again be careful with the decimal point!

8.3 ÷ 100

= 0.083

③ Write 0.16 as a fraction

There are 2 decimal places, so write it over 100

$$\frac{16}{100}$$

Now carefully simplify

$$\frac{16}{100} = \frac{8}{50} = \frac{4}{25}$$

④ Write $\frac{13}{20}$ as a decimal

We need to change the bottom of the fraction to 100, remembering to do the same to the top

$$\overset{\times 5}{\frac{13}{20} = \frac{65}{100}}\ \times 5$$

Divide the top of your fraction by 100 and you have your answer!

= 0.65

⑤ Write $\frac{5}{8}$ as a percentage

It's not easy to change this fraction over 100, so we must divide 5 by 8

5 ÷ 8

Use any method, but I do this: $= 8\overline{)5.000}$ with 0.625 above

0.625 is the answer as a decimal, so we must multiply by 100

0.625 × 100

= 62.5%

⑥ What is 12.5% as a fraction?

Start by writing the percentage over 100

$$\frac{12.5}{100}$$

We need to simplify, but the decimal point makes it hard. So why not multiply top and bottom by 2!

$$\times 2 \quad \frac{25}{200}$$

Now we can simplify as normal to get the answer:

$$\frac{25}{200} = \frac{5}{40} = \frac{1}{8}$$

Mathematics

Foundation

Unit 13

Simplifying Expressions

Note: To simplify an expression when adding or subtracting, draw boxes around all the LIKE TERMS and deal with each set of like terms on their own. To simplify an expression when multiplying, multiply the numbers together first, then the letters.

Adding and Subtracting

Example 1: Simplify $4m + 2p - m + 6p$

$$\boxed{4m}\boxed{+\ 2p}\boxed{-\ m}\boxed{+\ 6p}$$

First draw boxes around the like terms, making sure to include the sign in front

We have:

🟥 $4m - m = 3m$

🟦 $2p + 6p = 8p$

So: $4m + 2p - m + 6p = 3m + 8p$

Note: If you cannot see a sign in front of a term then just assume it is a **plus**

Example 2: Simplify $4t^2 - 5t - 2t - 3t^2$

We have:

$$\boxed{4t^2}\boxed{-\ 5t}\boxed{-\ 2t}\boxed{-\ 3t^2}$$

Be careful with the minus signs.
Remember: t and t^2 are different.

So: $4t^2 - 5t - 2t - 3t^2 = t^2 - 7t$

🟥 $4t^2 - 3t^2 = t^2$

🟦 $-5t - 2t = -7t$

Multiplying

Example 1: Simplify $5b \times 2c \times 3a$

Step 1: Multiply the numbers together first

$$5 \times 2 \times 3 = 30$$

Step 2: Multiply the letters

$$b \times c \times a = abc$$

Step 3: Put them together

$$5b \times 2c \times 3a = 30abc$$

Example 2: Simplify $4r \times -3p \times 3r \times q$

Step 1: Multiply the numbers together first, be careful with the negatives

$$4 \times -3 \times 3 \times 1 = -36$$

Note: There is no number in front of the q, which means it is a 1

Step 2: Multiply the letters

$$r \times p \times r \times q = pqrr = pqr^2$$

Remember: If you multiply something by itself it means you are squaring it

Step 3: Put them together

$$4r \times -3p \times 3r \times q = -36pqr^2$$

Mathematics

Foundation

Unit 13

Expanding Pairs of Single Brackets

When we expand pairs of single brackets, we separate the question into two parts, work each part out separately, then combine and simplify the answers.

$$3(5a-2) + 2(2a+4)$$

$3(5a-2) = 15a-6$

$2(2a+4) = 4a+8$

$$15a-6+4a+8 = 19a+2$$

Example 1: $6(x+4)+2(x-7)$

Separate into two parts:

$$6(x+4) + 2(x-7)$$

$6(x+4) = 6x+24$

$2(x-7) = 2x-14$

Combine and simplify:

$6x+24+2x-14 = 8x+10$

Example 2: $5(x-2)-3(x+1)$

Separate into two parts:

$$5(x-2) - 3(x+1)$$

$5(x-2) = 5x-10$

$-3(x+1) = -3x-3$

Combine and simplify:

$5x-10-3x-3 = 2x-13$

Example 3: $5(x-1)-2(x-3)$

Be careful with the minus signs

Separate into two parts:

$$5(x-1) - 2(x-3)$$

$5(x-1) = 5x-5$

$-2(x-3) = -2x+6$

Combine and simplify:

$5x-5-2x+6 = 3x+1$

Mathematics

Foundation

Unit 14

Substitution in Algebra

Substitution is where you are told the value of a letter and you substitute this into an expression or equation.

e.g. Find the value of $5x$ when $x = 7$, means 5 x x = 5 x 7 = 35.

- Always apply BIDMAS/BODMAS
- Use brackets for powers
- For fractions, work out the top and bottom separately.

Example 1: Evaluate (find the **value** of) the expressions, given that:

$$a = 2,\ \ b = 3,\ \ c = -5, d = -1$$

a) $5a = 5 \times 2$

$= \mathbf{10}$

b) $3b - 2c = 3 \times 3 - 2 \times (-5)$

$= 9 + 10$

$= \mathbf{19}$

c) $4b^2 + d = 4 \times 3^2 + (-1)$

$= 4 \times 9 - 1$

$= 36 - 1$

$= \mathbf{35}$

d) $3a^3 = 3 \times (2)^3$

$= 3 \times 8$

$= \mathbf{24}$

e) $\frac{5cd}{a+b} = \frac{5 \times (-5) \times (-1)}{2+3}$

$= \frac{25}{5}$

$= \mathbf{5}$

f) $c^2 + abd = (-5)^2 + 2 \times 3 \times (-1)$

$= 25 - 6$

$= \mathbf{19}$

Example 2: Evaluate (find the **value** of) the expressions, given that: (*calculator questions*)

$$a = 1.2,\ \ b = \frac{1}{9},\ \ c = -3.65$$

a) $4b - 6c + a^2 = 4 \times \frac{1}{9} - 6 \times (-3.65) + (1.2)^2$

$= \frac{4}{9} + 21.9 + 1.44$

$= \mathbf{23.78\dot{4}}$

b) $\sqrt{\frac{a+4c}{3b+c}} = \sqrt{\frac{1.2 + 4 \times (-3.65)}{3 \times \frac{1}{9} + (-3.65)}}$

$= \sqrt{\frac{-13.4}{-3.31\dot{6}}}$

$= \sqrt{4.0402010051}$

$= 2.0100251255$

Learn how to do these in one step using your *scientific* calculator.

Example 3: Use the formula $P = 5A - 6B$ to find the value of:

a) P when $A = 7$ and $B = -4$.

$P = 5A - 6B$

$P = 5 \times 7 - 6 \times (-4)$

$P = 35 + 24$

$P = 59$

b) A when $B = 3$ and $P = 37$

$P = 5A - 6B$

$37 = 5A - 6 \times 3$

$37 = 5A - 18$

$37 + 18 = 5A$

$55 = 5A$

$\frac{55}{5} = A$ $\qquad A = 11$

Mathematics

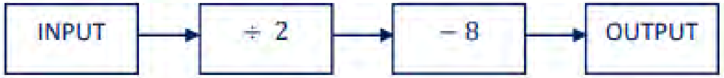
Foundation

Unit 14

Function Machines / Number Machines

Example 1:

A number machine is shown below.



a) Calculate the OUTPUT when the INPUT is 10.

Start with 10

Divide by 2 to give 5

Subtract 8 to give -3

The OUTPUT is -3

b) Calculate the INPUT when the OUTPUT is 7.

To find the INPUT from the OUTPUT use the inverse operations

(start at the end, go backwards through the number machine, do the opposite operation to the one given)

Start with 7

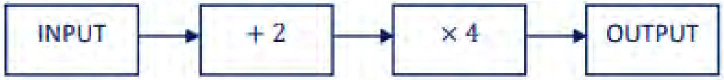
Add 8 (the opposite of −8) to give 15

Multiply by 2 (the opposite of ÷ 2) to give 30

The INPUT is 30

Example 2:

A number machine is shown below.



a) Calculate the OUTPUT when the INPUT is 3.

Start with 3

Add 2 to give 5

Multiply by 4 to give 20

The OUTPUT is 20

b) Calculate the INPUT when the OUTPUT is 16.

To find the INPUT from the OUTPUT use the inverse operations

(start at the end, go backwards through the number machine, do the opposite operation to the one given)

Start with 16

Divide by 4 (the opposite of × 4) to give 4

Subtract 2 (the opposite of +2) to give 2

The INPUT is 2

Mathematics

Foundation

Unit 15

Sequences

Sequence: A list which is in a particular order following a pattern.

Term: Each particular part of a sequence.

Term to term rule: This is the rule for finding the next pattern in a shape, or the next number in a sequence.

Finding the Term to Term Rule

Example 1: Some matchsticks have been laid to form patterns.
The sequence can be found by counting the matchsticks in each pattern.
What is the term to term rule for the pattern?

Pattern

3 matchsticks have been added to make the next pattern.

Term to term rule **Add 3 / + 3**

Example 2: Write the next two numbers in the sequence and describe in words the rule for continuing the sequence

35, 30, 25, 20, ..., ...

The numbers in the sequence are getting smaller which implies either subtracting a number or dividing by a number.

Goes down by the same number each time

So, the next two numbers in the sequence are 15 and 10 (20 - 5, 15 - 5)

The rule is: subtract 5 / - 5 / subtract 5 from the previous term

Example 3: Describe the term to term rule for continuing the sequence

3, 9, 27, 81, ...

The numbers in the sequence are getting bigger which implies either adding a number or multiplying by a number.

Does **not** go up by the same number each time, so look at multiplying

Multiplies by the same number each time

The rule is: multiply by 3 / x 3 / multiply the previous term by 3

Mathematics

Foundation

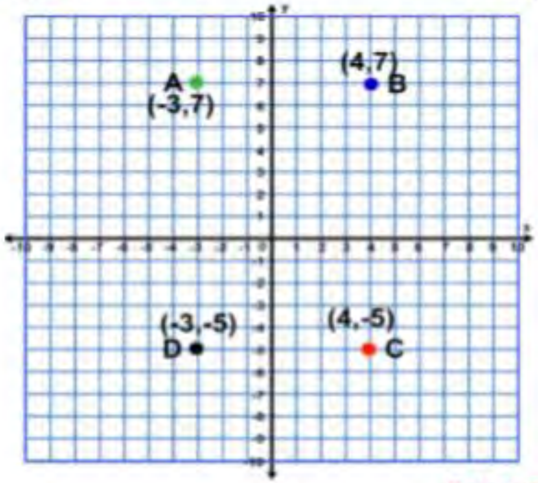
Unit 17

Plotting Coordinates

Example: Plot the points $A(-3, 7)$, $B(4, 7)$, $C(4, -5)$, and $D(-3, -5)$.

Point A has coordinates $(-3, 7)$, to plot A go across to -3 (left) and then up to 7.

Point B has coordinates $(4, 7)$, to plot B go across to 4 (right) and then up to 7.

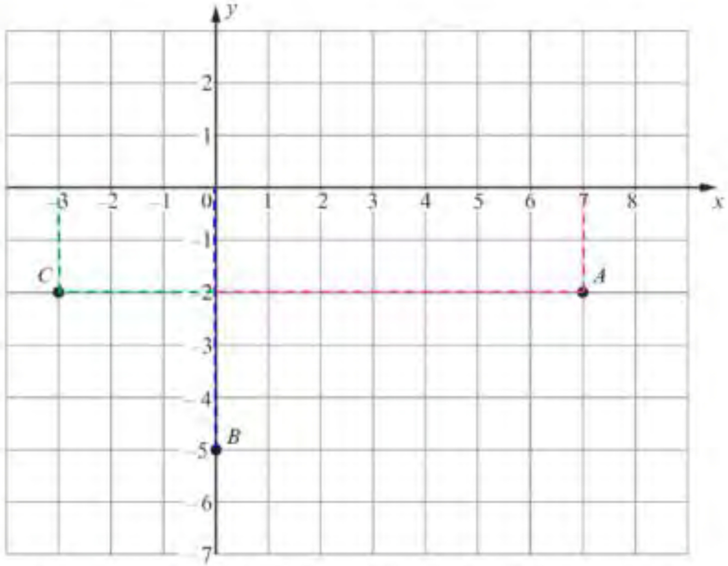


Point D has coordinates $(-3, -5)$, to plot D go across to -3 (left) and then down to -5.

Point C has coordinates $(4, -5)$, to plot C go across to 4 (right) and then down to -5.

Reading Coordinates

Example: Write down the coordinates of points A, B, and C.



Remember, read the x-coordinate first, then the y-coordinate. Write the coordinates in a bracket separated by a comma.

To get to A you go across to 7 and down to -2.
The coordinates of A are $(7, -2)$.

To get to B you stay at 0, and go down to -5.
The coordinates of B are $(0, -5)$.

To get to C you go across to -3 and down to -2.
The coordinates of C are $(-3, -2)$.

Mathematics

Foundation

Unit 19

3-D Shapes

Properties of 3-D Shapes

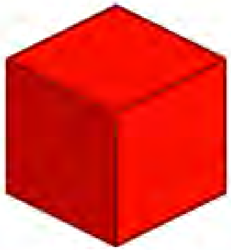
Vertex - the angular points of the shape (the corners)

Edge - where 2 faces meet.

2 or more corners are called **vertices**.

Face - the flat surface of the shape

Different Types of 3-D Shapes and their Properties



Cube

A cube has:

6 **equal** square faces,

12 edges,

and 8 vertices.

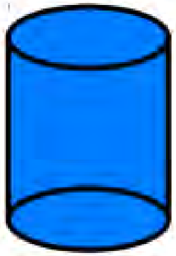
Cuboid

A cuboid has:

6 faces,

12 edges,

and 8 vertices.



Cylinder

A cylinder has:

3 faces,

2 edges,

and 0 vertices.

Cone

A cone has:

2 faces,

1 edge,

and 1 vertex.

Sphere

A sphere has:

1 face,

1 edge,

and 0 vertices.

Mathematics

Foundation

Unit 21

Basic Probability

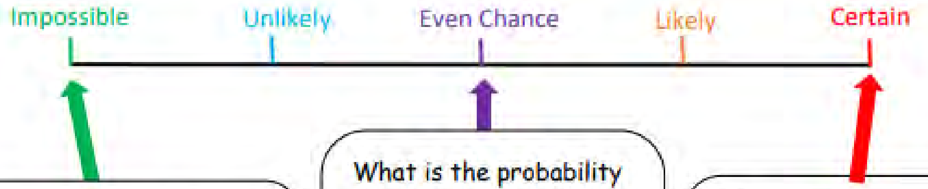
Probability is the likelihood that an event will occur.

Probabilities are always written as fractions, decimals, or percentages.

Probabilities have values between 0 and 1.

Probability scale

Probabilities can be described using words, and represented on a probability scale



What is the probability that Winter follows Summer?
Impossible

What is the probability that if you toss a coin it will land on heads?
Even Chance (the coin is equally likely to land on heads or tails)

What is the probability that Christmas will be on the 25th of December?
Certain

Probabilities can also be described using numbers

Impossible	Unlikely	Even	Likely	Certain
0		1/2		1
0		0.5		1.0
0%		50%		100%

Example: A box contains the following 8 cards. They are identical except for the numbers written on them.

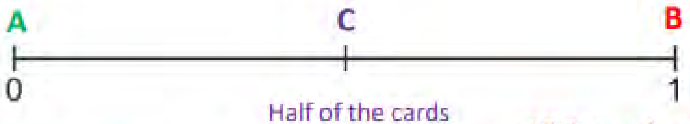
5 6 6 6 7 7 7 7

One card is chosen at random from the box. On the probability scale shown below, mark the points A, B, and C.

A is the probability that the chosen card has the number 3 on it

B is the probability that the chosen card has a number greater than 2 on it

C is the probability that the chosen card has a number less than 7 on it



There are no 3's on the cards so the probability is 0 (impossible)

Half of the cards have a number less than 7 so the probability is a half (even chance)

All the cards are greater than 2 so the probability is 1 (certain)

Mathematics

Foundation

Unit 23

Solving Simple Equations

The aim of solving an equation is to find the value of the unknown which makes the equation balance, e.g. equation: $x - 5 = 3$, solution: $x = 8$, because $8 - 5 = 3$.

There are different methods you can use to solve equations using your knowledge of inverse operations.

An operation is a mathematical process such as adding, multiplying, or squaring, etc.

An inverse operation is the process of reversing the operation (the opposite process).

For example, when adding, the inverse operation would be subtracting, when multiplying the inverse operation would be division and so on.

Here are the main inverse operations you need to know:

$+ \rightleftarrows -$

Addition is the opposite of subtracting.
Subtracting is the opposite of adding.
They are inverse operations.

$\times \rightleftarrows \div$

Multiplication is the opposite of division. Division is the opposite of multiplication
They are inverse operations.

Mathematics

Foundation

Unit 23

Method 1: Using our knowledge of inverse operations we can rearrange the equation to get the letter (this is often x) on its own. Many teachers say this is called the "Change the side, change the sign" method.

Golden Rule: When rearranging an equation and moving a term over the equals sign to the **opposite side** it changes to the **opposite sign** (the inverse). For example, '+3' becomes '-3', or '÷4' become 'x4'.

Note: The subject term is the letter used in the equation.

Step 1: Get rid of any square root signs by squaring both sides. Clear any fractions by cross-multiplying up to every other term. Multiply out any brackets.

Step 2: Collect all subject terms on one side of the equals sign and all non-subject terms on the other. Remembering the rule "change sides, change sign" (you most often see the letters on the left-hand side and numbers on the right).

Step 3: Simplify like terms on each side of the equation.

Step 4: If you are left with a number multiplied by your subject term equals something ($Ax = B$ where A and B are numbers and x is the subject term), then to get the subject term on its own, move the number over the other side of the equals sign remembering to change its sign to the opposite sign (the inverse) which in this case is from a multiply to a divide ($Ax = B$ becomes $x = \frac{B}{A}$).

Check your answer using substitution to make sure you are right.

Example 1: $p + 7 = 32$

$$p \boxed{+7} = 32$$

Move the $+7$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$p = 32 - 7$$

$$p = 25$$

Example 2: $r - 12 = 36$

$$r \boxed{-12} = 36$$

Move the -12 over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$r = 36 + 12$$

$$r = 48$$

Example 3: $\frac{k}{5} = -1$

$$\frac{k}{\boxed{5}} = -1$$

Move the $\div 5$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$k = -1 \times 5$$

$$k = -5$$

Example 4: $3m = 18$

$$\boxed{3}m = 18$$

Remember, $3m$ means $3 \times m$

Move the $\times 3$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$m = \frac{18}{3}$$

$$m = 6$$

Mathematics

Foundation

Unit 30

Example 4: $24 - 3m = 6$

$$24 - 3m = 6$$

Move the $+24$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$-3m = 6 - 24$$

$$-3m = -18$$

Remember, even though it is a -3, it is being multiplied by the m, so the opposite / inverse operation is a divide

Move the $\times(-3)$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$m = \frac{-18}{-3}$$

$$m = 6$$

Example 5: $7y + 3 = 10y - 6$

$$7y + 3 = 10y - 6$$

Move the $+3$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7y = 10y - 6 - 3$$

$$7y = 10y - 9$$

Move the $+10y$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7y - 10y = -9$$

$$-3y = -9$$

Move the $\times(-3)$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$y = \frac{-9}{-3}$$

$$y = 3$$

Example 6: $5(x - 3) = 4(x + 2)$

$$5(x - 3) = 4(x + 2)$$

Expand the brackets on both sides

$$5x - 15 = 4x + 8$$

Move the -15 over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$5x = 4x + 8 + 15$$

$$5x = 4x + 23$$

Move the $+4x$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$5x - 4x = 23$$

$$x = 23$$

Mathematics

Foundation

Unit 24

Averages and Dispersion

The main averages are the **mean**, **mode** and **median**.
The range is not an average but a measurement of **spread of data**.
The smaller the range the more consistent the data.

The mean

The mean uses all the values in the data.

To calculate the mean:

- Add up all of the items
- Divide by how many items there are

The mode

The mode is the most common value that appears in the data and there can be more than one.

If all the values appear the same number of times, then **there is no mode**.

Ordering the numbers can be helpful.

The median

The median is the middle value in the sorted set of data. To calculate the median:

- List the values in order from smallest to largest (ascending order)
- Cross values off from each end to identify the middle value

If there are two numbers in the middle, we add them up and divide by 2 to find the middle of those two numbers.

The range

The range is found by calculating the difference between the highest and lowest value.

Example 1: Find the mean, mode, median, and range of the following set of numbers:

10, 2, 3, 5, 15, 19, 21, 5

Mean: $\frac{2+3+5+5+10+15+19+21}{8} = \frac{80}{8} = 10$

Mode: 5

Median: ~~2~~, ~~3~~, ~~5~~, 5, 10, ~~15~~, ~~19~~, ~~21~~ $\frac{5+10}{2} = 7.5$

Range: 21 - 2 = 19

Example 2: Finding the total when given the mean of a set of numbers

The mean of a set of 6 numbers is 5. What is the total of the 5 numbers?

Remember, to find the mean $\frac{total\ value\ of\ items}{number\ of\ items} = mean$

Therefore, to find the original total of the numbers we use

$$total\ value\ of\ items = mean \times number\ of\ items$$

$total\ value\ of\ items = 5 \times 6 = 30$

Mathematics

Foundation

Unit 32

Probability 2

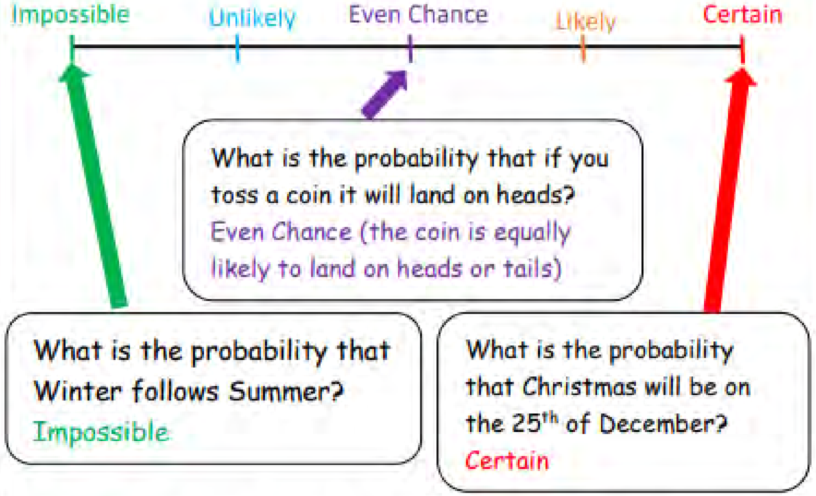
Probability is the likelihood that an event will occur.

Probabilities are always written as fractions, decimals, or percentages.

Probabilities have values between 0 and 1.

Probability scale

Probabilities can be described using words

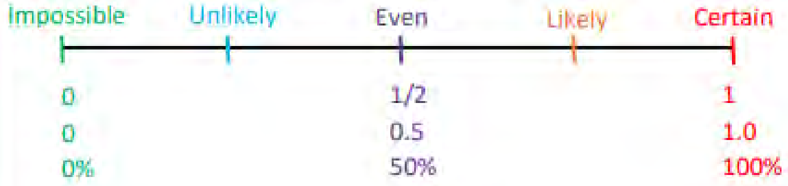


What is the probability that if you toss a coin it will land on heads?
Even Chance (the coin is equally likely to land on heads or tails)

What is the probability that Winter follows Summer?
Impossible

What is the probability that Christmas will be on the 25th of December?
Certain

Probabilities can also be described using numbers



The probability of an event happening can be found using:

$$P(\text{event happening}) = \frac{\text{number of ways the event could happen}}{\text{the total number of outcomes}}$$

Example: Find the probability of throwing an even number on a dice.

$$P(\text{even number}) = \frac{3}{6}$$

← Number of even numbers on a dice (2, 4, 6)

← **Total amount of numbers on a dice**

Example: What is the probability of picking a diamond from a full deck of cards?

$$P(\text{diamond}) = \frac{13}{52}$$

← Number of diamonds in a pack of cards

← **Total number of cards in a pack of cards**

The probability of an event **not** happening can be found using:

$$P(\text{event not happening}) = 1 - P(\text{event happening})$$

Example: What is the probability of **not** picking a diamond from a full deck of cards?

$$P(\text{not diamond}) = 1 - P(\text{diamond})$$

$$= 1 - \frac{13}{52} \qquad = \frac{39}{52}$$

Mathematics

Foundation

Unit 32

Listing Outcomes

You might be asked to list all the possible outcomes for two or more events.

Example: List all the 3-digit numbers that can be made using the digits 3, 6, and 9?

369 396 639 693 936 963

Example: A coin is flipped, and a dice is rolled. List all the possible outcomes.

A head on the coin → H 1

A 1 on the dice → H 1

H 1 H 2 H 3 H 4 H 5 H 6

T 1 T 2 T 3 T 4 T 5 T 6

A tail on the coin → T 6

A 6 on the dice → T 6

Sample Space Diagram

A sample space diagram is a way of showing multiple outcomes in one diagram.

Example: Two dice are thrown, and the numbers are multiplied together. The table below shows some of the possible outcomes.

Second Dice						
6	6	12	18	24	30	36
5	5	10	15	20	25	30
4	4	8	12	16	20	24
3	3	6	9	12	15	18
2	2	4	6	8	10	12
1	1	2	3	4	5	6
	1	2	3	4	5	6

First Dice

First dice x second dice
5 x 6 = 30

First dice x second dice
6 x 3 = 18

a) Complete the table to show **all** the possible outcomes.

b) What is the probability of getting an outcome that is an odd number? $P(\text{odd}) = \frac{9}{36} = \frac{1}{4}$

Number of outcomes that are odd numbers

Total number of outcomes

c) If the two dice were thrown a total of 60 times, how many times would you expect to get an outcome greater than 10? $P(6\text{ and }H) = 60 \times \frac{9}{36} = 15$ times

Number of times the dice are thrown

Probability of an odd number

Finding Missing Probabilities from a Table

Probabilities add up to 1, to find the missing probabilities add together the probabilities you are given and subtract them from 1.

Example: A biased spinner has 4 colours. The probability of the spinner landing on each colour is given below.

Colour	Red	Blue	Yellow	Green
Number of times	0.1	x	0.4	0.2

a) What is the probability of choosing a blue sweet?

$P(\text{Blue}) = 0.1 + 0.4 + 0.2 = 0.7$ ← Add the probabilities

$1 - 0.7 = 0.3$ ← Subtract them from 1

b) The spinner is spun 100 times. Calculate an estimate for the number of times the spinner will land on yellow.

Number of times the spinner is spun

Probability of a yellow

$P(\text{Yellow}) = 100 \times 0.4 = 40$ times

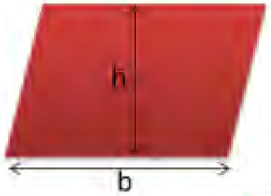
Mathematics

Foundation

Unit 22

Formulas for Area:

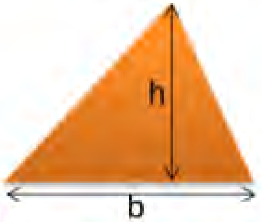
Parallelogram



A = b × h

Note: You must use the perpendicular height

Triangle

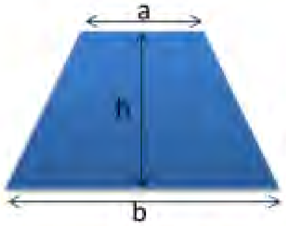


$A = \frac{b \times h}{2}$

Rectangle / Square

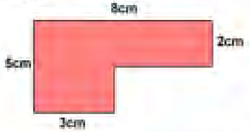
A = l × w

Trapezium



$A = \frac{(a + b) \times h}{2}$

Example 2: Find the perimeter and area of the compound shape



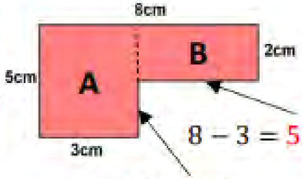
Area

Step 1: Split the shape into shapes that you can find the area of

Step 2: Find the missing lengths of sides

Step 3: Work out the area of each shape

Step 4: Work out the total area, remembering the units



$8 - 3 = 5cm$

$5 - 2 = 3cm$

$$\text{Area A} = (5 \times 3) \\ = 15cm^2$$

$$\text{Area B} = (2 \times 5) \\ = 10cm^2$$

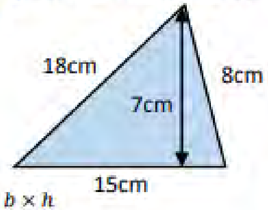
$$\text{Total area} = 15 + 10 \\ = 25cm^2$$

Perimeter

Add up all the outside edges

$$\text{Perimeter} = 3 + 5 + 8 + 2 + 5 + 3 \\ = 26cm$$

Example 3: Find the area of the triangle

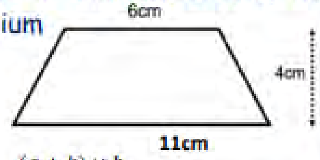


$$\text{Area} = \frac{b \times h}{2}$$

$$= \frac{15 \times 7}{2}$$

$$= 52.5cm^2$$

Example 4: Find the area of the trapezium



$$\text{Area} = \frac{(a + b) \times h}{2}$$

$$= \frac{(6 + 5) \times 4}{2}$$

$$= 22cm^2$$